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Design and control optimization of composite laminated truncated conical shells for minimum dynamic response including transverse shear deformation

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Abstract

The problem of minimizing the dynamic response of laminated truncated conical shells with minimum control force is presented. The total elastic energy of the shell is taken as a measure of the dynamic response which is formulated based on shear deformation theory. The ply thickness and fiber orientation angles are taken as optimization design variables. The Liapunov–Bellman theory is used to obtain explicit solutions for controlled deflections and closed loop control force. The present design and control optimization procedure is examined numerically for angle-ply, three-layer symmetric truncated conical shells with supported–supported, clamped–supported and clamped–clamped edges. The influences of the edges conditions, geometric and material parameters on the minimization process are illustrated.

Keywords: Design optimization; Active control; Design variables; Shear deformation theory; Laminated conical shell; Total elastic energy

1. Introduction

The constant demand for lighter and more efficient structural configurations has led the structural engineer to the use of new-made materials. At the same time, this demand has forced upon him very sophisticated methods of testing, analysis and design, as well as, of fabrication and manufacturing. The truncated conical shells constructed of fiber-reinforced laminated materials are widely used in aircraft, spacecraft, rocket and missile, which are frequently subjected to dynamic loads in service. In recent years, many studies have focused on the analysis of the structural response of these shells [1–6]. These studies indicated that the dynamic characteristics are of critical importance to the performance and safety of these structures [7].

Optimization is a central concept in the design of composite structures because of the adaptability of composite material to a given design situation. Design parameters such as layer thickness and ply orientations can be employed to minimize (or maximize) a certain effect to achieve an optimized structure with improved weight and stiffness characteristics. Research on the subject of structural optimization has been reported by many investigators [8–11]. In these studies, structural optimization of fiber-reinforced laminated truncated conical shells is performed to maximize their fundamental frequencies with respect to fiber orientations using a sequential linear programming method. Other structural optimization procedures for composite structures may be found in [12–15].

An effective means of suppressing excessive vibrations of the structures is by active control. Thus, there is need for new light materials possessing a high degree of flexibility and with low natural damping [16–18]. As a result, simultaneous design and control optimization approaches have been the main subject of several research studies [19–22]. A multiobjective optimization problems with constraints imposed on the relevant quantities are presented in, e.g., [23,24] for beams, plates and shells. The problems associated with optimization design and control of beams, plates and shells have been extensively studied, but relatively little attention has been devoted to the problems of laminated truncated conical shells, Also, many studies have indicated that the boundary conditions and the shear deformation have

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great effects on the design variables [15,20]. However, most previous studies are formulated based on classical theories for special cases of boundary conditions.

The subject of this work deals with the minimization of the dynamic response of composite laminated truncated conical shells with minimum expenditure of force using design and control optimization. The present formulation is based on a first-order shear deformation shell theory with various cases of boundary conditions. The dynamic response of the shell is expressed as the sum of the total elastic energy of the shell and a penalty functional involving closed loop control force. The ply thickness and fibers orientation angles are taken as design variables. Liapunov–Bellman theory is applied to obtain analytical solutions for controlled shell deflections and optimal control force. Various examples and numerical results are given to illustrate the effect of boundary conditions, material and geometric parameters on the minimization process.

2. Geometry of the shell and basic equations

Consider a composite laminated truncated circular conical shell with constant thickness h. Introduce the $x\theta z$ coordinates system located on the shell middle surface as shown in Fig. 1. The coordinate x is measured along the cone generator with origin at the boundary of the small base, the angle θ is the circumferential coordinate and z is the thickness coordinate. Let R_1 and R_2 denote the radii of the cone at the small and large edges, respectively. α is the semivertex angle of the cone and L is cone length along the meridional direction x. If R denotes the radius of the cone at any point along the merdional direction, then

$$R = R_1 + x\sin\alpha. \tag{1}$$

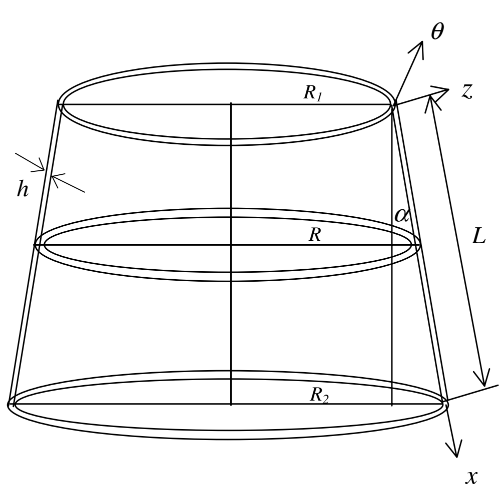

Fig. 1. The geometry and coordinates system of a truncated conical shell.

The outer surface of the conical shell is subjected to a distributed load $q(x,\theta,t)$ acting as control force. The initial conditions may be taken as

$$w(x,\theta,0) = A^*(x,\theta), \quad \dot{w}(x,\theta,0) = 0. \tag{2}$$

For the present formulation, a first-order shear deformation theory is used and accounting for a displacement field in the form:

$$u_x = u + z\psi, \quad u_\theta = v + z\phi, \quad u_z = w, \tag{3}$$

where (u_x, u_θ, u_z) are the displacements along x, θ and z directions, respectively, (u, v, w) are the displacements of a point on the mid-surface, and (ψ, ϕ) are the slopes of the normal to the mid-surface in the θz and xz surfaces.

The shell under consideration is composed of a finite number of orthotropic layers N. Let z_k and z_{k-1} be the top and bottom z-coordinates of the kth lamina. The stress–strain relations for a single lamina in a conical shell are given by

$$\begin{bmatrix} \sigma_x \\ \sigma_\theta \\ \tau_{x\theta} \\ \tau_{xz} \\ \tau_{\theta z} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{16} & 0 & 0 \\ C_{12} & C_{22} & C_{26} & 0 & 0 \\ C_{16} & C_{26} & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & C_{45} \\ 0 & 0 & 0 & C_{45} & C_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{x\theta} \\ \gamma_{xz} \\ \gamma_{\theta z} \end{bmatrix}, \tag{4}$$

where $(\sigma_x, \sigma_\theta, \tau_{x\theta})$ and $(\tau_{xz}, \tau_{\theta z})$ are the in-surface and transverse shear stresses, $(\varepsilon_x, \varepsilon_\theta, \gamma_{x\theta}, \gamma_{xz}, \gamma_{\theta z})$ are the strain components, and C_{ij} are the stiffness coefficients.

The kinematics relations in terms of the conical coordinates x, θ and z can be expressed as

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{x\theta} \\ \gamma_{xz} \\ \gamma_{\theta z} \end{bmatrix} = \begin{bmatrix} \partial_x & 0 & 0 \\ \sin\alpha/R & \partial_\theta/R & \cos\alpha/R \\ \partial_\theta/R & \partial_x - \sin\alpha/R & 0 \\ \partial_z & 0 & \partial_x \\ 0 & \partial_z - \cos\alpha/R & \partial_\theta/R \end{bmatrix} \begin{bmatrix} u_x \\ u_\theta \\ u_z \end{bmatrix}, \tag{5}$$

where $\partial_x \equiv \partial/\partial x$, $\partial_\theta \equiv \partial/\partial\theta$, $\partial_z \equiv \partial/\partial z$.

The equations governing the dynamic response of the conical shell are [2,5]:

$$\begin{aligned}
&\frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{x\theta}}{\partial\theta} + \frac{\sin\alpha}{R}(N_x - N_\theta) = \rho h\frac{\partial^2 u}{\partial t^2}, \\
&\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial N_\theta}{\partial\theta} + \frac{2\sin\alpha}{R}N_{x\theta} + \frac{Q_\theta\cos\alpha}{R} = \rho h\frac{\partial^2 v}{\partial t^2}, \\
&\frac{\partial Q_x}{\partial x} + \frac{1}{R}\frac{\partial Q_\theta}{\partial\theta} + \frac{Q_x\sin\alpha}{R} - \frac{N_\theta\cos\alpha}{R} + q = \rho h\frac{\partial^2 w}{\partial t^2}, \\
&\frac{\partial M_x}{\partial x} + \frac{1}{R}\frac{\partial M_{x\theta}}{\partial\theta} + \frac{\sin\alpha}{R}(M_x - M_\theta) - Q_x = \frac{\rho h^3}{12}\frac{\partial^2\psi}{\partial t^2}, \\
&\frac{\partial M_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial M_\theta}{\partial\theta} + \frac{2\sin\alpha}{R}M_{x\theta} - Q_\theta = \frac{\rho h^3}{12}\frac{\partial^2\phi}{\partial t^2},
\end{aligned} \tag{6}$$

where

$$\begin{bmatrix} N_x & M_x \\ N_\theta & M_\theta \\ N_{x\theta} & M_{x\theta} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_x \\ \sigma_\theta \\ \tau_{x\theta} \end{bmatrix} [1 \quad z]\,dz,$$

$$\begin{bmatrix} Q_x \\ Q_\theta \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \tau_{xz} \\ \tau_{\theta z} \end{bmatrix} dz. \tag{7}$$

In the present study, two types of boundary conditions, simply supported (S) and clamped (C) boundaries are taken, and they are described by [2,5]

For the simply supported boundary:

$$v = w = N_x = M_x = M_{x\theta} = 0. \tag{8}$$

For the clamped boundary:

$$u = v = w = \psi = M_{x\theta} = 0. \tag{9}$$

Setting $\alpha = 0$ and $\alpha = \pi/2$ in Eqs. (1), (5) and (6), we can obtain the governing equations corresponding to laminated cylindrical shells and annular plates, respectively.

3. The control objective

The present study aims to minimize the dynamic response of a laminated conical shell in a specified time $0 \leqslant t \leqslant \tau \leqslant \infty$ with the minimum possible expenditure of control force $q(x,\theta,t)$. The total elastic energy of the shell may be taken as a measure of the dynamic response so that the control objective may be written as

$$\begin{aligned} J(q,h_k,\gamma_k) = {} & \frac{1}{2}\int_0^\infty \int_0^L \int_0^{2\pi} \int_{-\frac{h}{2}}^{\frac{h}{2}} [\varepsilon_x\sigma_x + \varepsilon_\theta\sigma_\theta \\ & + \gamma_{x\theta}\tau_{x\theta} + \gamma_{xz}\tau_{xz} + \gamma_{\theta z}\tau_{\theta z}] R\,dz\,d\theta\,dx\,dt \\ & + \int_0^L \int_0^{2\pi} \left[\frac{1}{2}\rho^{(k)} \int_0^\infty \int_{-\frac{h}{2}}^{\frac{h}{2}} (\dot{u}_x^2 + \dot{u}_\theta^2 + \dot{u}_z^2)\,dz\,dt \right. \\ & \left. + \mu \int_0^\tau q^2(x,\theta,t)\,dt \right] R\,d\theta\,dx, \end{aligned} \tag{10}$$

where the weighting factor μ is a positive constant. The last term in (10) is a penalty functional involving the control function $q \in D^2$, D^2 denotes the set of all bounded square integrable functions on the domain of the solution.

The cost functional (10) of the present control problem depends on the distributed force $q(x,\theta,t)$, the ply thickness h_k and fiber orientation angles γ_k. Then the present optimal control problem can be reduced to determine the optimum variables q, h_k, and γ_k that minimize the cost functional (10).

4. Solution procedure

As usual the solution of the system of partial differential equations (6) with the boundary conditions (8) or (9) is sought in the separable form of double series in terms of the free vibration eigenfunctions of the shell. Then, the displacements functions (u,v,w,ψ,ϕ) and the closed loop control force q may be represented as

$$(u,v,w,\psi,\phi,q) = \sum_{m,n} (U_{mn}X_{,x}Y, V_{mn}XY_{,\theta}, W_{mn}XY, \Psi_{mn}X_{,x}Y, \Phi_{mn}XY_{,\theta}, Q_{mn}XY), \tag{11}$$

where U_{mn}, V_{mn}, W_{mn}, Ψ_{mn}, Φ_{mn} and Q_{mn} are unknown functions of time. The functions X and Y are continuous orthonormed functions, which satisfy at least the geometric boundary conditions, and represent approximate shapes of the deflected surface of the vibrating shell. These functions, for different cases of boundary conditions are given as:

For simply–simply supported (SS):

$$X = \sin\mu_m x, \quad \mu_m = m\pi/L, \quad Y = \cos n\theta.$$

For the clamped–clamped boundary (CC):

$$X = \sin\mu_m x - \sinh\mu_m x - \eta_m(\cos\mu_m x - \cosh\mu_m x),$$
$$Y = \cos n\theta,$$

$$\eta_m = (\sin\mu_m L - \sinh\mu_m L)/(\cos\mu_m L - \cosh\mu_m L),$$
$$\mu_m = (m+0.5)\pi/L.$$

For the clamped–simply boundary (CS):

$$X = \sin\mu_m x - \sinh\mu_m x - \eta_m(\cos\mu_m x - \cosh\mu_m x),$$
$$Y = \cos n\theta,$$

$$\eta_m = (\sin\mu_m L + \sinh\mu_m L)(\cos\mu_m L + \cosh\mu_m L)^{-1},$$
$$\mu_m = (m+0.25)\pi/L.$$

Using Eqs. (4), (5) and (7), we can get the governing equations (6) in terms of the displacements. For these equations, the in-plane inertia terms may be neglected. Substituting expressions (11) into the resulting equations and multiplying each equation by the corresponding eigenfunction, then integrating over the domain of solution, we obtain after some mathematical manipulations, the following time equations:

$$\begin{bmatrix} U_1 & V_1 & W_1 & \Psi_1 & \Phi_1 \\ U_2 & V_2 & W_2 & \Psi_2 & \Phi_2 \\ U_3 & V_3 & W_3 & \Psi_3 & \Phi_3 \\ U_4 & V_4 & W_4 & \Psi_4 & \Phi_4 \\ U_5 & V_5 & W_5 & \Psi_5 & \Phi_5 \end{bmatrix} \begin{bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \Psi_{mn} \\ \Phi_{mn} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \bar{\omega}\ddot{W}_{mn} - Q_{mn} \\ 0 \\ 0 \end{bmatrix}, \tag{12}$$

the coefficients U_i, V_i, W_i, Φ_i, Ψ_i and $\bar{\omega}$ are given in Appendix A. Solving the system (12), one gets an equation of the time-dependent functions W_{mn} and Q_{mn} only,

$$\ddot{W}_{mn} + \omega_{mn}^2 W_{mn} = l_{m,n} Q_{mn}, \tag{13}$$

where l_{mn} and ω_{mn}^2 are given in the Appendix B.

The six constructed series for u, v, w, ψ, ϕ and q will converge to their corresponding solutions if the small radius of the cone is not zero, i.e., $R_1 \neq 0$, which means that the conical shell is a truncated one [2]. A complete cone may be treated as a truncated cone with a very small radius at its apex. Also for all practical purposes there are no limitations on the other geometric parameters of the shell. Following previous analogous steps, we can get the objective functional (10) in the final form:

$$J = \frac{1}{2} \sum_{m,n} \int_0^\infty (k_1 W_{mn}^2 + k_2 \dot{W}_{mn}^2 + k_3 Q_{mn}^2)\,\mathrm{d}t, \tag{14}$$

where the coefficients k_i, $(i = 1, 2, 3)$ are given in Appendix C. Since the system of Eq. (13) is separable, hence the functional (14) depends only on the variables found in (m, n)th equation of the system. With the aid of this condition, the problem is reduced to a problem of analytical design of controllers [25,26] for every $m, n = 1, 2, \ldots, \infty$.

Now the optimal control and design problem is to find firstly, the control function $q_{mn}^{\mathrm{opt}}(t)$ that satisfies the conditions

$$J(q_{mn}^{\mathrm{opt}}) \leqslant J(q_{mn}) \quad \text{for all} \quad q_{mn}(t) \in D^2([0, \infty]),$$

that is

$$\min_{q_{mn}} J = \min \sum J_{mn} = \sum_{m,n} \min_{q_{mn} \in L^2} J$$

and, secondly, to find the optimum values of h_k and γ_k from the following minimization condition:

$$J(q_{mn}^{\mathrm{opt}}, h_k^{\mathrm{opt}}, \gamma_k^{\mathrm{opt}}) = \min_{h_k, \gamma_k} I(q_{mn}^{\mathrm{opt}}, h_k, \gamma_k),$$

$$\sum_k h_k = h, \quad 0 < \gamma_k < \pi/2.$$

For this problem, Liapunov–Bellman theory [26] may be used to determine the control force $q(x, \theta, t)$. This theory gives the necessary and sufficient conditions for minimizing the functional (10) in the form:

$$\min_q \left[\frac{\partial L_{mn}}{\partial W_{mn}} \dot{W}_{mn} + \frac{\partial L_{mn}}{\partial \dot{W}_{mn}} \ddot{W}_{mn} + \overline{J}_{mn} \right] = 0, \tag{15}$$

provided that the Liapunov function

$$L_{mn} = A_{mn} W_{mn}^2 + 2B_{mn} W_{mn} \dot{W}_{mn} + C_{mn} \dot{W}_{mn}^2, \tag{16}$$

is positive definite, i.e., $A_{mn} > 0$, $C_{mn} > 0$ and $A_{mn} C_{mn} > B_{mn}^2$, where $\overline{J}_{mn}$ is the integrand of Eq. (14). Using expressions (14)–(16), we can obtain the optimal control function in the form:

$$Q_{mn}^{\mathrm{opt}} = \frac{-l_{mn}}{k_3} (B_{mn} W_{mn} + C_{mn} \dot{W}_{mn}), \tag{17}$$

then, insert Eqs. (13), (14), (16) and (17) into (15) and equating the coefficients of W_{mn}^2, $\dot{W}_{mn}^2$ and $W_{mn}\dot{W}_{mn}$ by zeroes, the following system of equations is obtained

$$\begin{aligned} &l_{mn}^2 C_{mn}^2 - 2k_3 B_{mn} - k_3 k_4 = 0, \\ &l_{mn}^2 B_{mn}^2 + 2\omega_{mn}^2 k_3 B_{mn} - k_1 k_3 = 0, \\ &l_{mn}^2 B_{mn} C_{mn} + \omega_{mn}^2 k_3 C_{mn} - k_3 A_{mn} = 0. \end{aligned} \tag{18}$$

Under the condition that the Liapunov function is a positive definite, the solution of the system of nonlinear algebraic equations (18) may be obtained, then, using this solution into Eq. (13), one gets:

$$\ddot{W}_{mn} + \alpha_{mn} \dot{W}_{mn} + \beta_{mn}^2 W_{mn} = 0,$$

$$\alpha_{mn} = \frac{C_{mn} l_{mn}^2}{k_3}, \quad \beta_{mn}^2 = \omega_{mn}^2 + \frac{l_{mn}^2}{k_3} B_{mn},$$

the solution of this equation when $2\beta_{mn} > \alpha_{mn}$ is given by

$$W_{mn} = \mathrm{e}^{\frac{-\alpha_{mn} t}{2}} [\delta_{mn} \cos(\omega_{mn}^* t) + \tau_{mn} \sin(\omega_{mn}^* t)],$$

$$v_{mn} = \left(\beta_{mn}^2 - \frac{1}{4} \alpha_{mn}^2 \right)^{\frac{1}{2}},$$

where δ_{mn}, τ_{mn} are unknown coefficients which may be obtained from the initial conditions (2) by expanding them in a series. Thus, the controlled deflection solution takes the form:

$$W_{mn} = A^* \mathrm{e}^{\frac{-\alpha_{mn} t}{2}} \left(\cos(\omega_{mn}^* t) + \frac{\alpha_{mn}}{2\omega_{mn}^*} \sin(\omega_{mn}^* t) \right). \tag{19}$$

Insert expression (19) into (12), (14) and (17), we can get the controlled displacements, the total elastic energy and the optimal control force. Then, we complete the minimization process for the dynamic response of the shell by determining the optimal design of the shell using the design variables γ_k and h_k.

5. Numerical results and discussion

To study the influences of the boundary conditions, the material and geometric parameters on the control process, numerical results for maximum optimal control force q, central controlled deflection w and total elastic energy J are presented for symmetric angle-ply truncated conical shells with three types of boundary conditions, which are two ends clamped (denoted by CC), the small base clamped and the other base simply supported (denoted by CS) and two ends simply supported (denoted by SS). All layers of the laminate are assumed to be of the same orthotropic materials. A shear correction factors is taken to be 5/6. The plane reduced stress material stiffnesses Q_{ij} are given by

$$Q_{11}=\frac{E_1}{1-\nu_{12}\nu_{21}},\quad Q_{12}=\frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}},$$
$$Q_{22}=\frac{E_2}{1-\nu_{12}\nu_{21}},\quad Q_{44}=G_{23},\quad Q_{55}=G_{13},$$
$$Q_{66}=G_{12},\quad \nu_{ij}E_j=\nu_{ji}E_i,\quad (i,j=1,2),$$

where E_i are Young's moduli; ν_{ij} are Poisson's ratios and G_{ij} are shear moduli. In all calculations, unless otherwise stated, the following parameters are used:

$$h=2\text{ in.},\quad \rho=0.00012\text{ Ib.s}^2/\text{in.}^4,\quad R_2=20\text{ in.},$$
$$\mu=0.001,\quad A^*=10^3\ l\omega^{-2},\quad E_2=10^6\text{ psi},$$
$$E_1=25E_2,\quad G_{12}=G_{13}=0.5E_2,\quad G_{23}=0.2E_2,$$
$$\nu_{12}=0.25.$$

For the optimal design procedure, we consider angle-ply $(\gamma,0,\gamma)$ laminated shells with outer layers having the same thickness; and the optimization thickness variable r may be taken as the ratio of the outer layer thickness to the total shell thickness. All calculations in tables and figures are carried out at the midpoint of the shell for maximum amplitude of the deflection w and the control force q.

Table 1 contains numerical results for control force q, controlled deflection w and total elastic energy J for (45°, 0, 45°) truncated conical shells with SS edges. For these results, the radii R_1 and R_2 are taken to be fixed with some values of the length L and thickness h. Tables 2 and 3 present similar results for truncated conical shells with CS and CC edges. It is observed that, for conical shells controlled mechanically without optimal design, the elastic energy and deflections are very sensitive to the variations of the thickness and length in the three cases of edges conditions for all modes n. The total elastic energy and deflections rapidly increase with increasing the length and decreasing the thickness. This is because the large thin-walled structures have less resistance to the deformation than thick-walled structures. The annular plates (conical shell with $L/R_2=0.5$ or $\alpha=\pi/2$) have the least total energy, and consequently,

Table 1
Values of q, J and w for (45°, 0, 45°) shells, with SS, $R_1=10$, $R_2=20$, $E_1/E_2=25$, and $m=1$

n	L/R_2	$R_2/h=5$			$R_2/h=10$			$R_2/h=20$		
		q	$10J$	10^3w	q	$10J$	10^3w	q	$10J$	10^3w
2	0.5	84.835	13.533	8.2611	139.63	28.780	24.751	260.00	93.024	111.20
	1	109.20	46.491	14.045	164.97	83.943	35.715	240.99	150.91	89.845
	2	177.36	268.49	38.526	261.19	501.98	100.70	357.24	876.64	241.89
3	0.5	77.493	11.148	6.8090	123.05	21.640	18.643	222.98	62.383	75.142
	1	105.87	43.259	13.021	165.25	83.917	35.556	262.00	187.29	110.55
	2	156.90	206.46	30.421	256.78	492.41	101.16	409.28	1422.0	390.54
5	0.5	63.259	7.2506	4.4352	96.107	12.549	10.837	166.40	30.578	37.128
	1	83.413	25.884	7.8361	130.77	49.184	20.990	229.96	133.66	79.701
	2	96.207	70.330	10.614	156.54	148.11	31.453	284.83	471.39	138.49
9	0.5	44.220	3.4288	2.0984	65.007	5.4276	4.6923	105.36	10.843	13.226
	1	50.153	8.8838	2.7099	75.207	14.744	6.3499	126.87	32.661	19.823
	2	52.116	19.240	2.9329	78.803	32.564	7.0072	135.41	75.625	22.918

Table 2
Values of q, J and w for (45°, 0, 45°) shells, with CS, $R_1=10$, $R_2=20$, $E_1/E_2=25$, and $m=1$

n	L/R_2	$R_2/h=5$			$R_2/h=10$			$R_2/h=20$		
		q	$10J$	10^3w	q	$10J$	10^3w	q	$10J$	10^3w
2	0.5	110.58	22.596	9.5035	176.92	44.308	26.133	325.24	131.61	107.74
	1	151.77	89.265	18.655	231.03	163.05	47.957	342.15	301.86	124.41
	2	231.73	448.41	45.094	339.49	805.97	113.59	465.34	1356.1	264.77
3	0.5	103.66	19.708	8.3003	162.44	36.730	21.737	292.01	100.86	83.235
	1	146.25	82.178	17.156	227.74	157.32	46.185	360.80	345.46	141.52
	2	220.15	403.92	41.228	355.36	919.72	130.90	552.18	2353.2	452.42
5	0.5	88.363	14.090	5.9501	133.89	24.149	14.371	230.90	57.785	48.233
	1	119.57	53.262	11.164	187.09	100.59	29.694	326.54	267.18	110.10
	2	143.36	157.32	16.422	234.04	335.45	49.244	425.17	1080.1	218.76
9	0.5	64.123	7.2218	3.0564	94.215	11.417	6.8242	152.85	22.842	19.239
	1	74.659	19.775	4.1737	112.19	33.020	9.8388	190.16	74.195	31.139
	2	78.455	43.859	4.6267	119.19	75.168	11.193	206.54	179.09	37.548

Table 3
Values of q, J and w for (45°, 0, 45°) shells, with CC, $R_1 = 10$, $R_2 = 20$, $E_1/E_2 = 25$, and $m = 1$

n	L/R_2	$R_2/h = 5$			$R_2/h = 10$			$R_2/h = 20$		
		q	$10J$	10^3w	q	$10J$	10^3w	q	$10J$	10^3w
2	0.5	115.56	19.670	9.3888	175.90	34.331	23.012	305.55	85.652	79.848
	1	159.67	78.145	18.446	240.89	138.44	45.949	362.13	261.31	121.48
	2	242.64	386.99	44.190	356.51	689.94	110.55	494.36	1171.9	260.86
3	0.5	109.10	17.407	8.3095	164.69	29.744	19.974	282.50	71.191	66.727
	1	155.07	73.306	17.303	238.26	135.05	44.859	379.48	294.49	136.69
	2	230.50	347.54	40.103	368.21	756.45	122.14	574.47	1861.9	410.05
5	0.5	94.207	12.766	6.0927	140.11	20.969	14.118	233.91	46.059	43.538
	1	127.43	47.997	11.342	197.16	88.039	29.344	339.72	222.89	104.04
	2	152.97	141.49	16.602	247.51	293.54	48.498	447.19	907.38	207.85
9	0.5	68.856	6.6269	3.1580	100.70	10.365	6.9798	160.93	20.008	19.016
	1	80.054	18.043	4.2785	119.74	29.774	9.9716	200.81	65.011	30.705
	2	84.160	40.006	4.7369	127.21	67.671	11.313	218.47	156.99	36.982

they need the least control energy to reduce their vibrational response.

Tables 4–6 present numerical results for q, w and J for (45°, 0, 45°) conical shells with fixed big radius R_2 and shell length L, and with varying small radius R_1 in the three cases of boundary conditions. As mentioned before, the series solutions are convergent under the condition $R_1 \neq 0$. Therefore for complete cone, the small radius is taken as $R_1 = 10^{-6}$ in. Note that the effect of the small radius on the energy is more stronger in the range $0 < R_1/R_2 < 0.6$ than those cases in the range $0.6 < R_1/R_2 < 1$. Also, the elastic energy rapidly increases with increasing the small radius, so that, the complete cone ($R_1 \to 0$) has the least total energy, whereas, the cylindrical shell ($R_1 = R_2$) has the largest energy. Then, the control energy used in minimization process increases with increasing R_1. Also the results in Tables 4–6 confirm the previous discussion on the effects of edges conditions and thickness on the control process. In general, the results in Tables 1–6 show that all geometric parameters and the boundary conditions may play significant role in reducing the vibrational energy of the truncated conical shells.

Table 7 contains four groups of numerical results for optimum values of ply orientations γ_{opt}, optimum thickness ratio r_{opt} and controlled total elastic energy J_{opt} for some values of ratios $E_1/E_2, R_2/h, L/R_2$ and R_1/R_2. Note that, the optimum thickness ratio r_{opt} is constant ($r_{opt} = 0.5$) for all values of geometric and material parameters, and for all cases of edges conditions except the cases of long truncated conical shells with $L/R_2 \geqslant 3$. The optimum values of the orientation angle γ_{opt} have low sensitivity to the variation of the material and geometrical parameters, and to the different cases of edges conditions.

Figs. 2 and 3 include J-curves plotted against the ratios E_1/E_2 and R_2/h for SS truncated conical shells designed optimally by three methods, which are design

Table 4
Values of q, J and w for (45°, 0, 45°) shells, with SS, $R_2 = 20$, $E_1/E_2 = 25$, $L/R_2 = 2$ and $m = 1$

n	R_1/R_2	$R_2/h = 5$			$R_2/h = 10$			$R_2/h = 20$		
		q	$10J$	10^3w	q	$10J$	10^3w	q	$10J$	10^3w
2	0	124.04	79.538	17.405	181.35	135.13	41.545	253.93	222.34	95.529
	0.6	180.76	299.70	40.308	264.25	552.57	103.88	358.50	945.90	244.68
	1	184.91	401.00	43.682	264.67	703.68	107.17	352.63	1140.3	239.35
3	0	97.550	47.406	10.526	150.96	88.691	27.608	237.49	187.79	81.199
	0.6	165.87	250.43	34.515	271.38	608.54	116.67	425.95	1749.2	445.44
	1	189.81	431.31	47.392	303.62	1039.8	158.09	445.10	2579.1	518.69
5	0	59.379	16.592	3.7325	89.633	28.098	8.8903	148.95	61.040	26.960
	0.6	103.07	87.169	12.323	169.39	190.01	37.766	309.34	635.60	173.79
	1	128.50	177.62	20.029	216.67	431.63	68.137	389.64	1634.9	345.75
9	0	32.659	4.8260	1.0929	47.271	7.3130	2.3369	73.294	12.937	5.8102
	0.6	55.793	23.668	3.3818	85.079	40.946	8.2572	148.35	99.395	28.210
	1	70.114	47.887	5.4691	110.26	90.052	14.503	200.11	252.85	57.073

Table 5
Values of q, J and w for (45°, 0, 45°) shells, with CS, $R_2 = 20$, $E_1/E_2 = 25$, $L/R_2 = 2$ and $m = 1$

n	R_1/R_2	$R_2/h = 5$			$R_2/h = 10$			$R_2/h = 20$		
		q	$10J$	10^3w	q	$10J$	10^3w	q	$10J$	10^3w
2	0	195.04	219.08	30.504	292.88	403.23	78.778	417.73	721.41	196.09
	0.6	234.57	487.82	46.587	342.13	869.45	116.36	466.76	1447.0	268.27
	1	243.90	654.13	52.274	351.73	1143.3	127.96	474.04	1855.7	287.45
3	0	159.14	141.78	20.167	252.60	287.35	57.372	409.72	695.75	192.31
	0.6	228.19	462.99	44.753	366.96	1055.6	142.14	562.16	2636.5	478.70
	1	250.34	697.82	55.889	394.23	1552.6	172.86	575.27	3462.7	519.79
5	0	97.911	50.464	7.3078	150.56	90.074	18.400	257.81	216.91	62.223
	0.6	151.52	187.29	18.516	249.15	409.91	56.945	452.77	1364.9	260.39
	1	181.26	336.72	27.482	303.15	796.94	91.105	539.79	2841.7	437.65
9	0	53.920	14.622	2.1333	78.733	22.686	4.6767	125.04	42.792	12.453
	0.6	83.028	52.093	5.2068	127.05	90.965	12.832	222.74	224.80	44.618
	1	100.79	94.729	7.8251	158.27	177.49	20.676	286.54	494.75	80.793

Table 6
Values of q, J and w for (45°, 0, 45°) shells, with CC, $R_2 = 20$, $E_1/E_2 = 25$, $L/R_2 = 2$ and $m = 1$

n	R_1/R_2	$R_2/h = 5$			$R_2/h = 10$			$R_2/h = 20$		
		q	$10J$	10^3w	q	$10J$	10^3w	q	$10J$	10^3w
2	0	198.16	163.09	27.885	297.56	295.08	70.940	432.50	538.75	180.76
	0.6	246.41	428.56	45.978	360.63	759.06	114.25	497.33	1274.4	266.43
	1	260.36	614.40	53.250	377.46	1075.9	130.66	513.21	1763.9	297.10
3	0	158.31	100.96	17.562	247.86	194.93	47.741	405.88	462.23	157.84
	0.6	239.55	404.92	43.805	381.19	881.63	133.32	585.39	2108.4	434.24
	1	265.28	643.58	55.890	414.21	1374.0	166.35	605.04	2945.5	485.27
5	0	96.772	35.687	6.2964	146.48	61.047	15.208	246.42	137.82	48.360
	0.6	163.00	173.23	19.049	265.83	369.59	57.181	480.35	1181.0	252.14
	1	199.10	336.79	29.610	330.65	779.91	96.126	582.88	2637.9	440.26
9	0	53.295	10.401	1.8467	77.218	15.822	3.9703	120.15	28.354	10.050
	0.6	89.951	49.023	5.4419	137.06	84.665	13.267	238.60	204.73	45.171
	1	112.48	98.050	8.7084	176.44	183.27	22.958	318.83	508.03	89.246

Table 7
Optimum values γ_{opt}, r_{opt} and J_{opt} for $(\gamma, 0, \gamma)$ truncated conical shells $m = 1$, $n = 2$

		SS			CS			CC		
		γ_{opt}	r_{opt}	J_{opt}	γ_{opt}	r_{opt}	J_{opt}	γ_{opt}	r_{opt}	J_{opt}
Group 1: $R_2/h = 10$, $L/R_2 = 1$, $R_1/R_2 = 0.5$										
E_1/E_2	5	48.5°	0.5	24.0	51.2°	0.5	44.0	50°	0.5	36.5
	15	49.7°	0.5	9.51	53.1°	0.5	17.8	53°	0.5	15.5
	40	50.2°	0.5	4.04	53.9°	0.5	7.51	54.4°	0.5	6.73
Group 2: $E_1/E_2 = 25$, $L/R_2 = 1$, $R_1/R_2 = 0.5$										
R_2/h	5	51.8°	0.5	3.65	53.1°	0.5	6.81	53.6°	0.5	6.10
	15	50.3°	0.5	8.17	54°	0.5	15.4	54.3°	0.5	13.7
	25	50.6°	0.5	11.5	54.4°	0.5	21.6	54.9°	0.5	19.4
Group 3: $E_1/E_2 = 25$, $R_2/h = 10$, $R_1/R_2 = 0.5$										
L/R_2	1	50°	0.5	6.04	53.6°	0.5	11.4	53.9°	0.5	10.1
	2	34°	0.5	47.1	39.3°	0.5	69.4	42.2°	0.5	57.2
	3	50.3°	0.33	197	33.2°	0.5	292	36.4°	0.5	219
	4	71.3°	0.4	405	56°	0.32	781	49.2°	0.35	597
Group 4: $E_1/E_2 = 25$, $R_2/h = 10$, $L/R_2 = 1$										
R_1/R_2	0.2	49.9°	0.5	5.70	51.1°	0.5	12.3	50.9°	0.5	9.87
	0.6	51.1°	0.5	6.29	54.5°	0.5	11.7	54.9°	0.5	10.6
	1	56.4°	0.5	8.38	58.2°	0.5	15	58.5°	0.5	14.5

optimization over the ply thickness only, design optimization over the orientation angles only and design optimization over both the ply thicknesses and orientation angles. These curves indicate that all cases of design optimization considerably reduce the total elastic energy of the conical shells. But the optimization over both the ply thicknesses and orientation angles is the most efficient for all values of the geometric and material ratios. Moreover these optimal designs are more required for thin conical shells with large ratios of L/R_2 and for cylindrical shells ($R_1 = R_2$). This discussion remains true in the three considered cases of edges conditions (see Figs. 4 and 5). Figs. 6 and 7 display q-curves plotted against the ratios L/R_2, R_1/R_2, E_1/E_2 and R_2/h. These curves indicate that the design optimization over both the ply thickness and orientation angles is the most efficient in reducing the needed control force. The behavior of the energy J and the control force q with time t is displayed in Fig. 8 for four cases of design and control optimization. Two cases are for uncontrolled shells ($q = 0$) with (or without) optimal design. The other two cases are for shells controlled mechanically ($q \neq 0$) with (or without) optimal design. These cases show that the optimal design procedure reduces significantly the level of the energy without any external control force, whereas, the mechanical control is less active in reducing the level of the energy at the start of the process, but it helps in damping the energy rapidly. In addition the

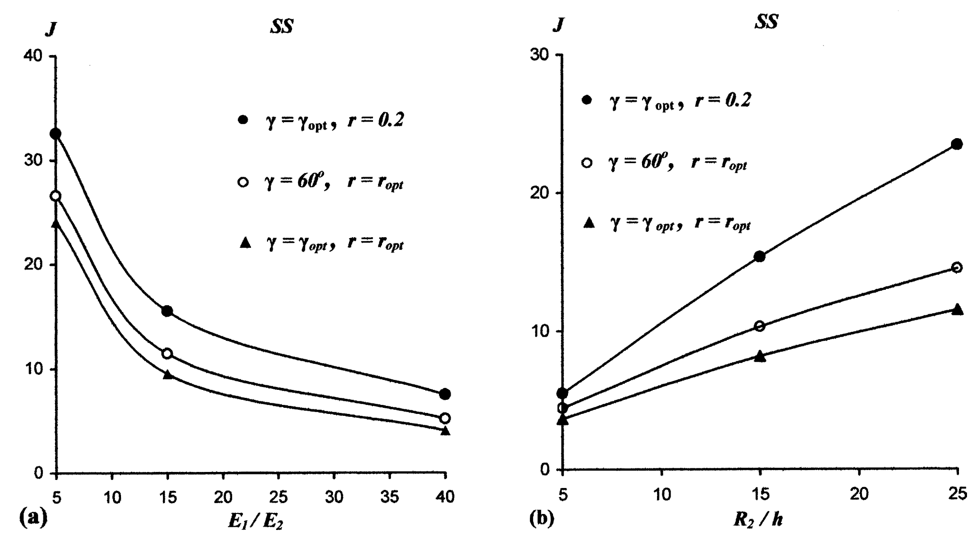

Fig. 2. Curves of J plotted against E_1/E_2 and R_2/h for $(\gamma, 0, \gamma)$ SS conical shells: (a) $L/R_2 = 1$, $R_2/h = 10$, $R_1/R_2 = 0.5$, (b) $E_1/E_2 = 25$, $L/R_2 = 1$, $R_2/R_2 = 0.5$.

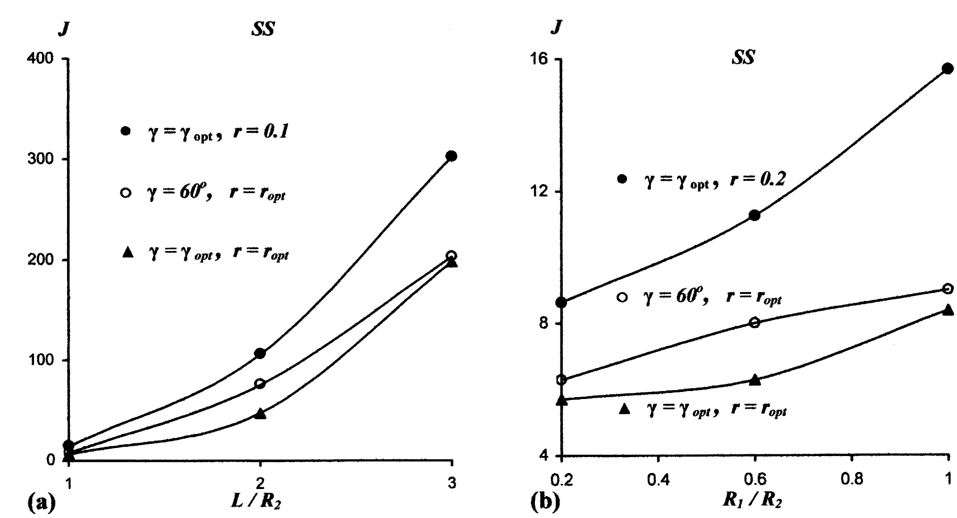

Fig. 3. Curves of J plotted against R_2/h and L/R_2 for $(\gamma, 0, \gamma)$ SS shells: (a) $E_1/E_2 = 25$, $R_2/h = 10$, $R_1/R_2 = 0.5$, (b) $E_1/E_2 = 25$, $L/R_2 = 1$, $R_2/h = 10$.

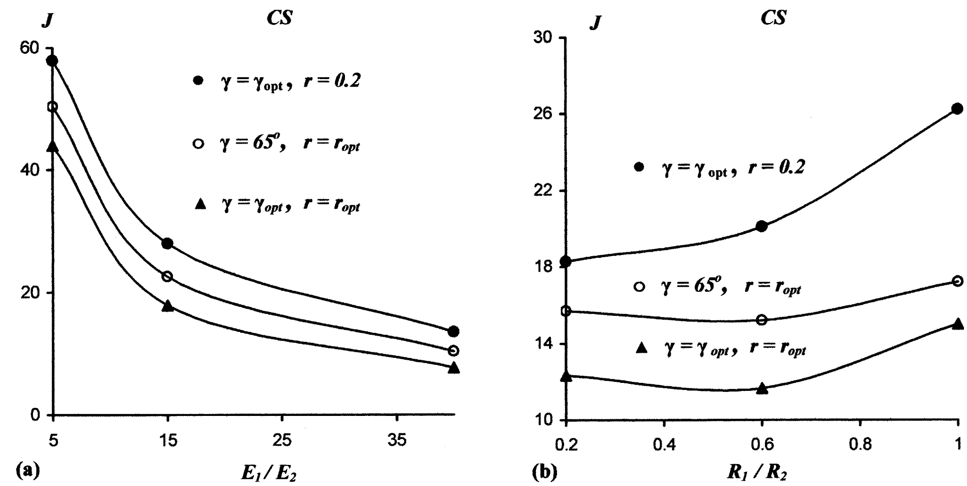

Fig. 4. Curves of J plotted against E_1/E_2 and R_1/R_2 for $(\gamma, 0, \gamma)$ CS shells: (a) $L/R_2 = 1$, $R_2/h = 10$, $R_1/R_2 = 0.5$, (b) $E_1/E_2 = 25$, $L/R_2 = 1$, $R_2/h = 10$.

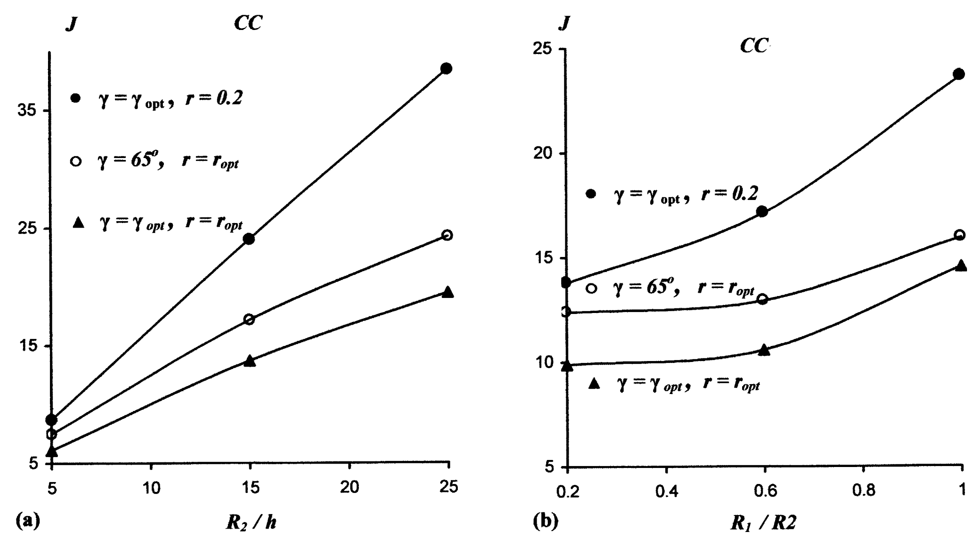

Fig. 5. Curves of J plotted against R_2/h and R_1/R_2 for $(\gamma, 0, \gamma)$ CC shells: (a) $E_1/E_2 = 25$, $L/R_2 = 1$, $R_1/R_2 = 0.5$, (b) $E_1/E_2 = 25$, $L/R_2 = 1$, $R_2/h = 10$.

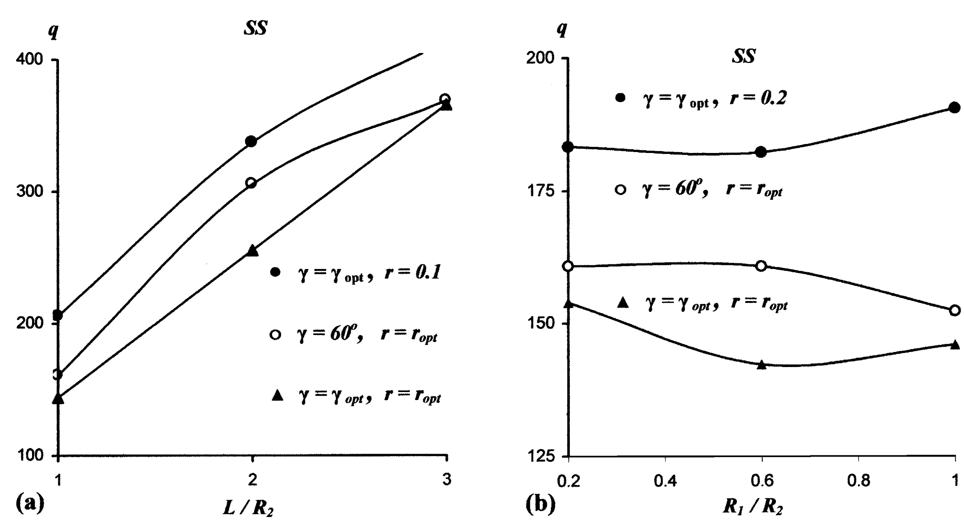

Fig. 6. Curves of q plotted against L/R_2 and R_1/R_2 for $(\gamma, 0, \gamma)$ SS shells: (a) $E_1/E_2 = 25$, $R_2/h = 10$, $R_1/R_2 = 0.5$, (b) $E_1/E_2 = 25$, $L/R_2 = 1$, $R_2/h = 10$.

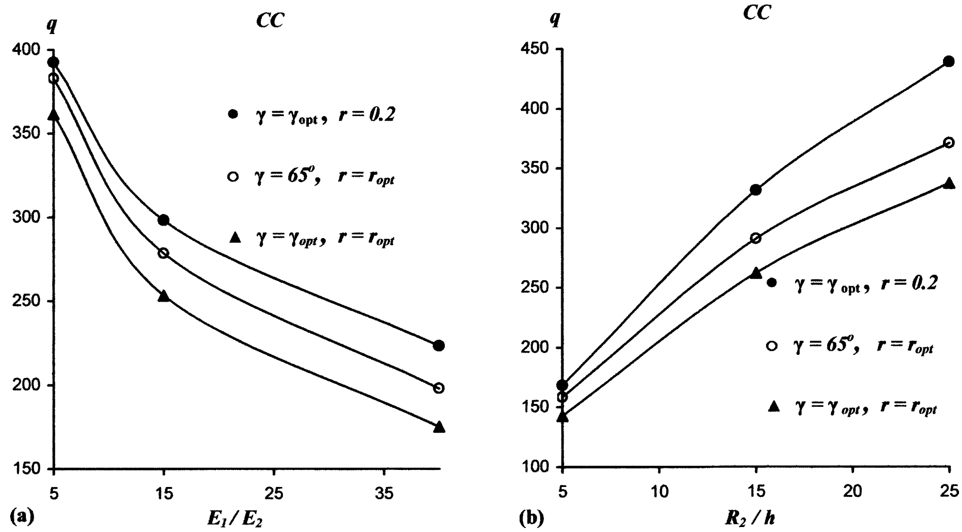

Fig. 7. Curves of q plotted against E_1/E_2 and R_2/h for CC conical $(\gamma, 0, \gamma)$ shells: (a) $L/R_2 = 1$, $R_2/h = 10$, $R_1/R_2 = 0.5$, (b) $E_1/E_2 = 25$, $L/R_2 = 1$, $R_2/R_2 = 0.5$.

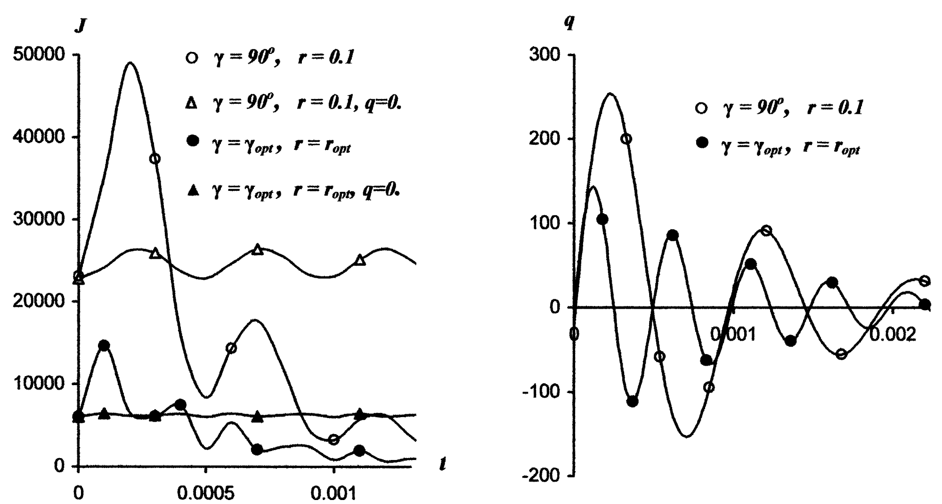

Fig. 8. J- and q-curves plotted against time t for conical shells with SS, $E_1/E_2 = 25$, $L/R_2 = 1$, $R_2/h = 10$ and $R_1/R_2 = 0.5$.

simultaneous design and control optimization is very active in reducing and damping the energy with minimum possible expenditure of control force in least possible period of the time.

6. Conclusion

Design and control optimization procedure is used to minimize the dynamic response of laminated truncated conical shells with minimum possible expenditure of force. The total elastic energy of the shell is taken as the measure of the dynamic response. The present problem is formulated based on shear deformation shell theory. The optimum ply thicknesses, optimum ply orientation angles and optimum closed loop control force are determined for shells with some cases of edges conditions. The present results indicate that all geometric and material parameters of the shells can play important role in the minimizing process. The design and control optimization procedure is very active in minimizing and damping the dynamic response rapidly in all considered cases of edges conditions. Also, it reduces considerably the needed control energy, as well as, the time of the minimization process.

Appendix A

$$U_1 = A_{66}e_1 + \overline{A}S^2e_2 + A_{11}(e_5 + Se_9),$$
$$V_1 = (A_{12} + A_{66})e_7 + (\overline{A} - A_{66})Se_3,$$
$$W_1 = \overline{A}CSe_4 + A_{12}Ce_8,$$

$$\Psi_1 = B_{66}e_1 + \overline{B}S^2e_2 + B_{11}(e_5 + Se_9),$$
$$\Phi_1 = (B_{12} + B_{66})e_7 + (\overline{B} - B_{66})Se_3,$$
$$W_2 = (A_{22} + A_{44})Ce_{12},$$
$$U_2 = (A_{22} + 2A_{66})Se_{13} + (A_{12} + A_{66})e_{16},$$
$$V_2 = A_{22}e_{11} - (2A_{66}S^2 + A_{44}C^2)e_{12} + A_{66}(e_{14} + Se_{18}),$$
$$\Psi_2 = (B_{22} + 2B_{66})Se_{13} + (B_{12} + B_{66})e_{16},$$
$$\Phi_2 = B_{22}e_{11} - 2B_{66}S^2e_{12} + B_{66}e_{14} + A_{44}Ce_{17} + B_{66}Se_{18},$$
$$U_3 = -C(A_{22}Se_4 + A_{12}e_{24}),$$
$$V_3 = -C(A_{22} + A_{44})e_{19},$$
$$W_3 = A_{55}(Se_{10} + e_{21}) + A_{44}e_{19} - A_{22}C^2e_{20},$$
$$\Psi_3 = A_{55}(Se_{10} + e_{21}) - C(B_{22}Se_4 + B_{12}e_{24}),$$
$$\Phi_3 = -B_{22}Ce_{19} + A_{44}e_{23}, \quad W_5 = B_{22}Ce_{12} - A_{44}e_{17},$$
$$U_4 = B_{66}e_1 + \overline{B}S^2e_2 + B_{11}(e_5 + Se_9),$$
$$V_4 = (\overline{B} - B_{66})Se_3 + (B_{12} + B_{66})e_7,$$
$$W_4 = \overline{B}CSe_4 - A_{55}e_6 + B_{12}Ce_8,$$
$$\Psi_4 = D_{66}e_1 - \overline{D}S^2e_2 + D_{11}e_5 - A_{55}e_6 + D_{11}Se_9,$$
$$\Phi_4 = (\overline{D} - D_{66})Se_3 + (D_{12} + D_{66})e_7, \quad V_5 = \Phi_2,$$
$$U_5 = (B_{22} + 2B_{66})Se_{13} + (B_{12} + B_{66})e_{16},$$
$$\Psi_5 = (D_{22} + 2D_{66})Se_{13} + (D_{12} + D_{66})e_{16},$$
$$\Phi_5 = D_{22}e_{11} - 2D_{66}S^2e_{12} + D_{66}e_{14} + A_{44}e_{15} + D_{66}Se_{18},$$
$$\overline{A} = A_{12} - A_{22}, \quad \overline{B} = B_{12} - B_{22}, \quad \overline{D} = D_{12} - D_{22},$$
$$\bar{\varpi} = -2I_1e_{22}, \quad S = \sin\alpha, \quad C = \cos\alpha,$$

Appendix B

$$l = \frac{1}{\bar{\varpi}}, \quad \omega^2 = \frac{1}{\Delta\bar{\varpi}}(\Delta_1U_3 + \Delta_2V_3 + \Delta_3\Psi_3 + \Delta_4\Phi_3 - \Delta W_3),$$

$$\Delta = \begin{vmatrix} U_{1mn} & V_{1mn} & \Psi_{1mn} & \Phi_{1mn} \\ U_{2mn} & V_{2mn} & \Psi_{2mn} & \Phi_{2mn} \\ U_{4mn} & V_{4mn} & \Psi_{4mn} & \Phi_{4mn} \\ U_{5mn} & V_{5mn} & \Psi_{5mn} & \Phi_{5mn} \end{vmatrix},$$

$$\Delta_1 = \begin{vmatrix} W_{1mn} & V_{1mn} & \Psi_{1mn} & \Phi_{1mn} \\ W_{2mn} & V_{2mn} & \Psi_{2mn} & \Phi_{2mn} \\ W_{4mn} & V_{4mn} & \Psi_{4mn} & \Phi_{4mn} \\ W_{5mn} & V_{5mn} & \Psi_{5mn} & \Phi_{5mn} \end{vmatrix},$$

$$\Delta_2 = \begin{vmatrix} U_{1mn} & W_{1mn} & \Psi_{1mn} & \Phi_{1mn} \\ U_{2mn} & W_{2mn} & \Psi_{2mn} & \Phi_{2mn} \\ U_{4mn} & W_{4mn} & \Psi_{4mn} & \Phi_{4mn} \\ U_{5mn} & W_{5mn} & \Psi_{5mn} & \Phi_{5mn} \end{vmatrix},$$

$$\Delta_3 = \begin{vmatrix} U_{1mn} & V_{1mn} & W_{1mn} & \Phi_{1mn} \\ U_{2mn} & V_{2mn} & W_{2mn} & \Phi_{2mn} \\ U_{4mn} & V_{4mn} & W_{4mn} & \Phi_{4mn} \\ U_{5mn} & V_{5mn} & W_{5mn} & \Phi_{5mn} \end{vmatrix},$$

$$\Delta_4 = \begin{vmatrix} U_{1mn} & V_{1mn} & \Psi_{1mn} & W_{1mn} \\ U_{2mn} & V_{2mn} & \Psi_{2mn} & W_{2mn} \\ U_{4mn} & V_{4mn} & \Psi_{4mn} & W_{4mn} \\ U_{5mn} & V_{5mn} & \Psi_{5mn} & W_{5mn} \end{vmatrix}.$$

$$(e_1, e_2, e_3, e_4, e_5) = \int_0^{2\pi}\int_0^L (\overline{X}_{,x}Y_{,\theta\theta}, \overline{X}_{,x}Y, \overline{X}Y_{,\theta\theta}, \overline{X}Y, \widehat{X}_{,xxx}Y)X_{,x}Y\,\mathrm{d}x\,\mathrm{d}\theta, \quad \overline{X} = X/R,$$

$$(e_6, e_7, e_8, e_9, e_{10}) = \int_0^{2\pi}\int_0^L (\widehat{X}_{,x}Y, X_{,x}Y_{,\theta\theta}, X_{,x}Y, X_{,xx}Y, XY)X_{,x}Y\,\mathrm{d}x\,\mathrm{d}\theta, \quad \widehat{X} = XR,$$

$$(e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, , e_{16}, e_{17}, e_{18}) = \int_0^{2\pi}\int_0^L (\overline{X}Y_{,\theta\theta\theta}, \overline{X}Y_{,\theta}, \overline{X}_{,x}Y_{,\theta}, \widehat{X}_{,xx}Y_{,\theta}, \widehat{X}Y_{,\theta}, X_{,xx}Y_{,\theta}, XY_{,\theta}, X_{,x}Y_{,\theta})XY_{,\theta}\,\mathrm{d}x\,\mathrm{d}\theta,$$

$$(e_{19}, e_{20}, e_{21}, e_{22}, e_{23}, e_{24}, e_{25}) = \int_0^{2\pi}\int_0^L (\overline{X}Y_{,\theta\theta}, \overline{X}Y, \widehat{X}_{,xx}Y, \widehat{X}Y, XY_{,\theta\theta}, X_{,xx}Y, X_{,xx}Y_{,\theta\theta})XY\,\mathrm{d}x\,\mathrm{d}\theta,$$

$$(e_{26}, e_{27}, e_{28}, e_{29}, e_{30}) = \int_0^{2\pi}\int_0^L (X_{,x}\overline{X}_{,x}Y_{,\theta}^2, X\overline{X}Y_{,\theta\theta}^2, X_{,x}\widehat{X}_{,x}Y_{,\theta}^2, X_{,xx}\widehat{X}_{,xx}Y^2, X_{,x}^2Y_{,\theta}^2)\,\mathrm{d}x\,\mathrm{d}\theta,$$

$$I_n = \sum_{k=1}^{N}\int_{z_{k-1}}^{z_k}\rho^{(k)}z^{n-1}\,\mathrm{d}z, \quad (n = 1, 2, 3), \quad (A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{N}\int_{z_{k-1}}^{z_k}C_{ij}^{(k)}(1, z, z^2)\,\mathrm{d}z, \quad (i, j = 1, 2, 4, 5, 6).$$

Appendix C

$$k_1 = (k_{55}L_4 + k_{15}L_1 + k_{25}L_2 + k_{35} + k_{45}L_3)L_4 + (k_{44}L_3 + k_{14}L_1 + k_{34} + k_{24}L_2)L_3 + (k_{22}L_2 + k_{23} + k_{12}L_1)L_2 + (k_{11}L_1 + k_{13})L_1 + k_{33},$$

$$k_2 = \frac{1}{2}e_{22},$$

$$k_3 = I_1e_{22} + (I_1L_1^2 + 2I_2L_1L_3 + I_3L_3^2)e_{23} + (I_1L_2^2 + 2I_2L_2L_4 + I_3L_4^2)e_{31},$$

$$k_{11} = \frac{1}{2}(A_{22}S^2e_2 + 2A_{12}Se_9 + A_{66}e_{26} + A_{11}e_{29}),$$

$$k_{12} = A_{22}Se_3 - A_{66}Se_{13} + A_{12}e_{25} + A_{66}e_{30},$$

$$k_{13} = A_{22}SCe_4 + A_{12}Ce_{24},$$

$$k_{14} = B_{22}S^2e_2 + 2B_{12}Se_9 + B_{66}e_{26} + B_{11}e_{29},$$

$$k_{35} = A_{44}e_{17} + B_{22}Ce_{19},$$

$$k_{15} = B_{22}Se_3 - B_{66}Se_{13} + B_{12}e_{25} + B_{66}e_{30},$$

$$k_{24} = B_{22}S^2e_2 + 2B_{12}Se_9 + B_{66}e_{26} + B_{11}e_{29},$$

$$k_{22} = \frac{1}{2}(A_{44}C^2e_{12} + A_{66}S^2e_{12} - 2A_{66}Se_{18} + A_{22}e_{27} + A_{66}e_{28}),$$

$$k_{23} = -A_{44}Ce_{12} + A_{22}Ce_{19},$$

$$k_{25} = +B_{66}S^2e_{12} - A_{44}Ce_{17} - 2B_{66}Se_{18} + B_{22}e_{27} + B_{66}e_{28},$$

$$k_{33} = \frac{1}{2}(A_{55}e_6 + A_{44}e_{12} + A_{22}C^2e_{20}),$$

$$k_{34} = B_{22}CSe_4 + A_{55}e_6 + B_{12}Ce_{24},$$

$$k_{55} = \frac{1}{2}(A_{44}e_{15} + D_{66}S^2e_{12} - 2D_{66}Se_{18} + D_{22}e_{27} + D_{66}e_{28}),$$

$$k_{44} = \frac{1}{2}(D_{22}e_2S^2 + A_{55}e_6 + 2D_{12}Se_9 + D_{66}e_{26} + A_{11}e_{29}),$$

$$L_1 = -\Delta_1/\Delta, \quad L_2 = -\Delta_2/\Delta,$$

$$k_{45} = D_{22}Se_3 + D_{66}Se_{13} + D_{12}e_{25} + D_{66}Se_{13},$$

$$L_3 = -\Delta_3/\Delta, \quad L_4 = -\Delta_4/\Delta.$$

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